Metric Propositional Neighborhood Logics: Expressiveness, Decidability, and Undecidability

Davide Bresolin[1], Dario Della Monica[2], Valentin Goranko[3], Angelo Montanari[2], and Guido Sciavicco[4]

[1] University of Verona, Italy - davide.bresolin@univr.it
[2] University of Udine, Italy - {dario.dellamonica,angelo.montanari}@dimi.uniud.it
[3] Technical University of Denmark, Denmark - vfgo@imm.dtu.dk
[4] University of Murcia, Spain - guido@um.es

Interval temporal logics provide a natural framework for reasoning about interval temporal structures over linearly (or partially) ordered domains. They take time intervals, instead of time points, as the primitive ontological entities over which truth of formulae is defined. Modal operators of interval logics correspond to various relations between pairs of intervals. In particular, Halpern and Shoham's modal logic of time intervals (HS) [8] features a set of modal operators that makes it possible to express all Allen's interval relations [1]. Interval-based formalisms have been extensively used in various areas of computer science and artificial intelligence. However, most of them are subjected to severe syntactic and semantic restrictions that considerably weaken their expressive power. Interval temporal logics relax these restrictions, thus allowing one to cope with much more complex application domains. Unfortunately, many of them, including HS and the majority of its fragments, turn out to be undecidable [3]. One of the few cases of decidable interval logic is Propositional Neighborhood Logic (PNL), interpreted over various classes of interval structures (all, dense, and discrete linear orders, integers, natural numbers) [7]. PNL is a fragment of HS with only two modalities, corresponding to Allen's relations *meets* and *met by*. The satisfiability problem for PNL has been addressed by Bresolin et al. in [5]. NEXPTIME-completeness with respect to various classes of linearly ordered domains has been proved via a reduction to the satisfiability problem for the two-variable fragment of first-order logic for binary relational structures over ordered domains [14].

Various metric extensions to point-based temporal logics have been proposed in the literature. They include Alur and Henzinger's Timed Propositional Temporal Logic [2], Montanari and de Rijke's two-sorted metric temporal logics [12], Hirshfeld and Rabinovich's Quantitative Monadic Logic of Order [9], and Owakine and Worrell's Metric Temporal Logic [15], which refines and extends Koymans' Metric Temporal Logic [11]. Little work has been done in the interval setting. Among the few contributions, we mention Kautz and Ladkin's extension of Allen's Interval Algebra with a notion of distance [10]. The most important quantitative interval logic is Duration Calculus (DC) [17], based on Moszkowski's ITL [13]. DC is quite expressive, but undecidable with respect to most classes of interval structures. A number of variants and fragments of DC have been proposed in the literature to model and to reason about real-time processes and systems. Many of them recover decidability by imposing semantic restrictions, such as the *locality* principle, that essentially reduce the logic to a point-based one.

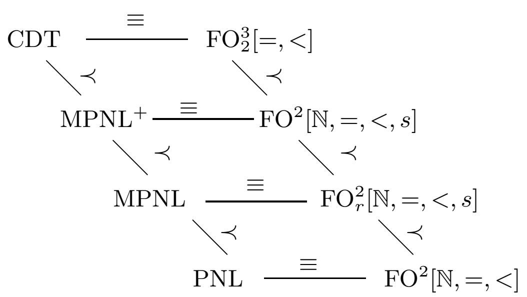

Fig. 1. Expressive completeness results for interval logics.

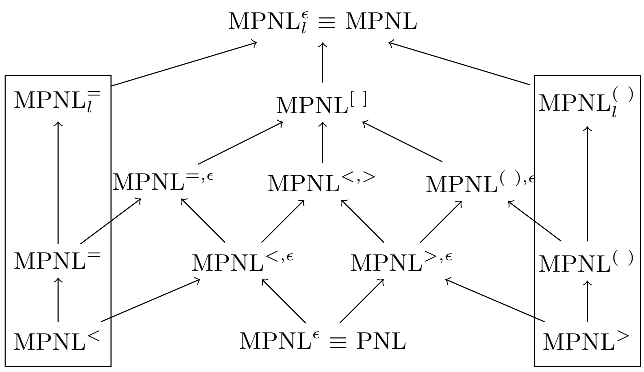

Fig. 2. Relative expressive power of MPNL fragments. Fragments inside the boxes are not expressive enough to capture PNL.

In this work, we present a family of non-conservative metric extensions of PNL, which allow one to express *metric properties* of interval structures over natural numbers. Two different ways of extending PNL are considered: adding a metric dimension to PNL modalities or introducing integer constraints on interval length.

The most expressive extension is *Metric PNL* (MPNL, for short). MPNL features a family of special atomic propositions of the form $\mathsf{len}_{\sim k}$, with $\sim \in \{<, \leq, =, >, \geq\}$ and $k \in \mathbb{N}$. They can be used to constrain the length of the intervals over which they are evaluated. The right-neighbor (future) fragment of MPNL has been introduced and studied in [6]. Decidability of its satisfiability problem is proved by showing a bounded model property with respect to ultimately periodic models. In addition, an EXPSPACE procedure for satisfiability checking and a proof of EXPSPACE-hardness by a reduction from the exponential corridor tiling problem are given. We prove the decidability of full MPNL by devising a double exponential time nondeterministic procedure for checking satisfiability (see [4]). The exact complexity of the problem is still an open question (a lower bound immediately follows from the result for its future fragment). Then, we prove the expressive completeness of MPNL with respect to $\mathrm{FO}^2_r[\mathbb{N}, =, <, s]$, a syntactic fragment of the two-variable fragment $\mathrm{FO}^2[\mathbb{N}, =, <, s]$ of FOL with equality, linear order, successor, and any family of uninterpreted binary relations, interpreted on natural numbers. We also define an extension of MPNL, called MPNL^+, which is expressively complete with respect to full $\mathrm{FO}^2[\mathbb{N}, =, <, s]$. Undecidability of full $\mathrm{FO}^2[\mathbb{N}, =, <, s]$ can be proved by a reduction from the octant tiling problem (undecidability of MPNL^+ immediately follows). Expressive completeness results for interval temporal logics are summarized in Figure 1. In [16], Venema defines a proper extension of HS, called CDT, and he proves its expressive completeness with respect to $\mathrm{FO}^3_2[=, <]$ (the three-variable fragment of FOL, with equality, linear order, and any family of uninterpreted binary relations, where at most two variables occur free). In [5], Bresolin et al. prove the expressive completeness of PNL with respect to $\mathrm{FO}^2[=, <]$ (the two-variable fragment of FOL with equality, linear order, and any family of uninterpreted binary relations). The results about MPNL and its undecidable extension MPNL^+ provide a finer characterization of the decidability/undecidability border with respect to the class of FOL fragments. Finally, we classify the considered metric extensions of PNL with respect to their relative expressive power. The outcomes of such a classification are summarized in Figure 2, where an arrow from L_1 to L_2 means that L_1 is strictly less expressive than L_2 (according to the notation given in [4], superscripts collect the modal operators featured by the logics, while the subscript l identifies all and only the logics with atomic propositions for length constraints).

References

[1] J. F. Allen. Maintaining knowledge about temporal intervals. *Communications of the ACM*, 26(11):832–843, 1983.

[2] R. Alur and T.A. Henzinger. A really temporal logic. *Journal of the ACM*, 41:181–204, 1994.

[3] D. Bresolin, D. Della Monica, V. Goranko, A. Montanari, and G. Sciavicco. Decidable and Undecidable Fragments of Halpern and Shoham's Interval Temporal Logic: Towards a Complete Classification. In I. Cervesato, H. Veith, and A. Voronkov, editors, *Proc. of the 15th International Conference on Logic for Programming, Artificial Intelligence, and Reasoning*, volume 5330 of *LNCS*, pages 590–604. Springer, 2008.

[4] D. Bresolin, D. Della Monica, V. Goranko, A. Montanari, and G. Sciavicco. On the expressiveness and (un)decidability of metric propositional neighborhood logics. In *Proc. of the 19th European Conference on Artificial Intelligence, in press*, 2010.

[5] D. Bresolin, V. Goranko, A. Montanari, and G. Sciavicco. Propositional interval neighborhood logics: Expressiveness, decidability, and undecidable extensions. *Annals of Pure and Applied Logic*, 161(3):289 – 304, 2009.

[6] D. Bresolin, V. Goranko, A. Montanari, and G. Sciavicco. Right propositional neighborhood logic over natural numbers with integer constraints for interval lengths. In *Proc. of the 7th IEEE Conf. on Software Engeneering and Formal Methods*, pages 240–249, 2009.

[7] V. Goranko, A. Montanari, and G. Sciavicco. Propositional interval neighborhood temporal logics. *Journal of Universal Computer Science*, 9(9):1137–1167, 2003.

[8] J. Halpern and Y. Shoham. A propositional modal logic of time intervals. *Journal of the ACM*, 38(4):935–962, 1991.

[9] Y. Hirshfeld and A.M. Rabinovich. Logics for real time: Decidability and complexity. *Fundamenta Informaticae*, 62(1):1–28, 2004.

[10] H.A. Kautz and P.B. Ladkin. Integrating metric and qualitative temporal reasoning. In *Proc. of the 9th National Conference on Artificial Intelligence (AAAI)*, pages 241–246, 1991.

[11] R. Koymans. Specifying real-time properties with metric temporal logic. *Real Time Systems*, 2(4):255–299, 1990.

[12] A. Montanari and M. de Rijke. Two-sorted metric temporal logic. *Theoretical Computer Science*, 183(2):187–214, 1997.

[13] B. Moszkowski. *Reasoning about digital circuits*. Tech. rep. stan-cs-83-970, Dept. of Computer Science, Stanford University, Stanford, CA, 1983.

[14] M. Otto. Two variable first-order logic over ordered domains. *Journal of Symbolic Logic*, 66(2):685–702, 2001.

[15] J. Ouaknine and J. Worrell. Some recent results in metric temporal logic. In *Proc. of the 6th International Conference on Formal Modelling and Analysis of Timed Systems*, pages 1–13, 2008.

[16] Y. Venema. A modal logic for chopping intervals. *Journal of Logic and Computation*, 1(4):453–476, 1991.

[17] C. Zhou and M. R. Hansen. *Duration Calculus: A Formal Approach to Real-Time Systems.* EATCS: Monographs in Theoretical Computer Science. Springer, 2004.